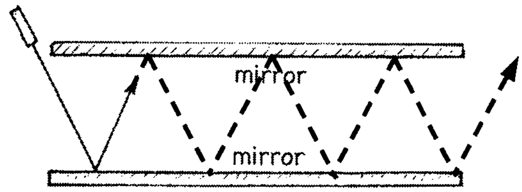
28-1. (a) The angle between the reflected ray and the normal is also $\boldsymbol{\theta}$.

(b) The angle between the reflected ray and the incident ray is $\boldsymbol{2\theta}$.

28-2. (a) From the sketch, the beam would reflect **7 times**.

(b) For ideal mirrors, **the strength of the beam would be undiminished**. (For real mirrors, there is a small loss upon each reflection.)



28-3. (a)The incident beam will now make an angle of $(30° + \alpha)$ with the normal, and the reflected beam will also make an angle of $(30° + \alpha)$ with the normal. So the angle of reflection has **changed by $\boldsymbol{\alpha}$.**

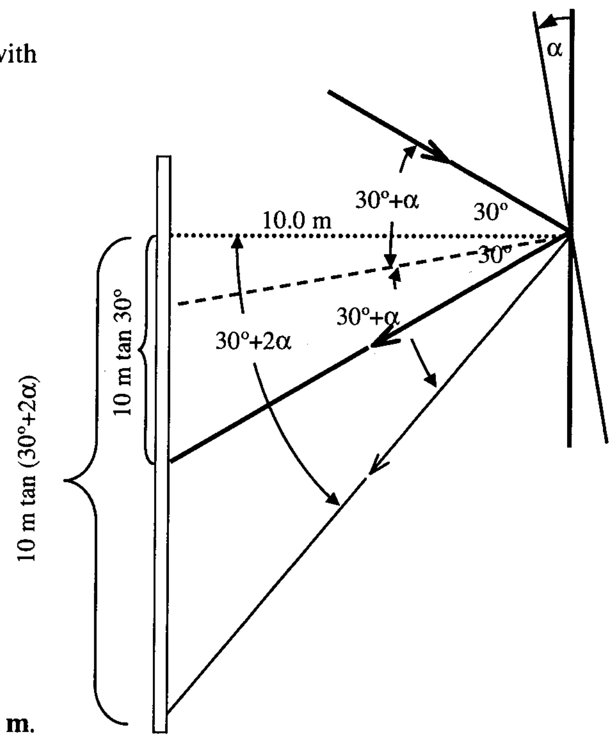
(b) From the diagram, the original beam was deflected by $(10.0\,\text{m})\tan 30°$ relative to where the normal to the mirror hits the screen. When the mirror is rotated by an angle α, the reflected beam makes an angle of $(30° + 2\alpha)$ with the *original* normal, and strikes the screen a distance $(10.0\text{ m})\tan(30° + 2\alpha)$ relative to where the *original* normal hits the screen. So the change in the position of the dot on the screen is

$10.0\,\text{m}\tan(30° + 2\alpha) - 10.0\,\text{m}\tan 30°$

$= 10.0\,\text{m}\left(\tan(30° + 2\alpha) - \tan 30°\right).$

With $\alpha = 2°$, the spot on the screen will move

$10.0\text{ m}\left[\tan\left(30° + 2(2°)\right) - \tan 30\right]° = \mathbf{0.97\ m}.$



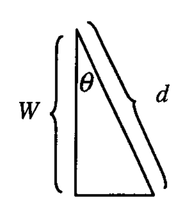
28-4. (a) Your image also "approaches" the mirror at a speed v from the other side of the mirror. Your speed relative to your image is $\boldsymbol{2v}$.

(b) Likewise, your image "walks away" from the mirror at the same speed as you do, so you and your image recede from each other at a relative speed of $\boldsymbol{2v}$.

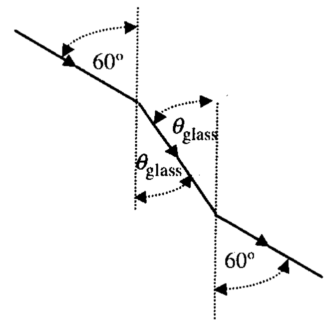
28-5. (a) The ball would make an angle $\boldsymbol{\theta}$ to the normal.

(b) $d = ?$ From the diagram, $\cos\theta = \dfrac{W}{d} \;\Rightarrow \boldsymbol{d = \dfrac{W}{\cos\theta}}$.

(c) "English" (for a pool ball) means "spin". If the ball has some spin on it friction with the side of the pool table could give the ball a push or pull in some direction beyond a neat reflection.



28-6. (a) $\theta_{glass} = ?$ From $n_{air} \sin\theta_{air} = n_{glass} \sin\theta_{glass} \quad \Rightarrow \sin\theta_{glass} = \dfrac{n_{air}\sin\theta_{air}}{n_{glass}}$



$$\Rightarrow \theta_{glass} = \sin^{-1}\left(\frac{n_{air}\sin\theta_{air}}{n_{glass}}\right) = \sin^{-1}\left(\frac{1(\sin 60^\circ)}{1.5}\right) = \mathbf{35^\circ}.$$

(b) Since the glass surfaces are parallel, θ_{glass} will be the same for the refracted beam entering the glass and again when the beam approaches the second glass surface. Since θ_{glass} is the same, θ_{air} will be the same as well, **60°**.

28-7. (a) $$\frac{v_{\text{light in glass}}}{v_{\text{light in diamond}}} = \frac{\left(\dfrac{c}{n_{glass}}\right)}{\left(\dfrac{c}{n_{diamond}}\right)} = \frac{n_{diamond}}{n_{glass}} = \frac{2.42}{1.50} = \mathbf{1.61}.$$

(b) Since the light reflects, the angle of incidence and the angle of reflection are **equal**.

28-8. (a) $$v_{red} - v_{blue} = \frac{c}{n_{red}} - \frac{c}{n_{blue}} = c\left(\frac{n_{blue} - n_{red}}{n_{blue}n_{red}}\right) = 3.00\times10^{8}\,\tfrac{\text{m}}{\text{s}}\left(\frac{1.675 - 1.638}{1.675(1.638)}\right) = \mathbf{4.05\times10^{6}\,\tfrac{m}{s}}.$$

(b) $t = ?$ From $v = \dfrac{x}{t} \quad \Rightarrow t = \dfrac{x}{v_{red}} = \dfrac{x}{\left(\dfrac{c}{n_{red}}\right)} = \dfrac{n_{red}x}{c} = \dfrac{1.638\,(0.0300\text{ m})}{3.00\times10^{8}\,\frac{\text{m}}{\text{s}}} = \mathbf{1.64\times10^{-10}\,s}.$

28-9. (a) $\theta_{\text{blue light, glass}} = ?$ From $n_{air}\sin\theta_{air} = n_{glass}\sin\theta_{glass} \quad \Rightarrow \sin\theta_{glass} = \dfrac{n_{air}\sin\theta_{air}}{n_{glass}}$

$$\Rightarrow \theta_{\text{blue light, glass}} = \sin^{-1}\left(\frac{n_{air}\sin\theta_{air}}{n_{\text{blue light, glass}}}\right) = \sin^{-1}\left(\frac{1(\sin 36.00^\circ)}{1.675}\right) = \mathbf{20.54^\circ}.$$

(b) $$\theta_{\text{red light, glass}} = \sin^{-1}\left(\frac{n_{air}\sin\theta_{air}}{n_{\text{red light, glass}}}\right) = \sin^{-1}\left(\frac{1(\sin 36.00^\circ)}{1.638}\right) = \mathbf{21.03^\circ}.$$

28-10. (a) $\theta_{\text{red light, water}} = ?$ From $n_{air}\sin\theta_{air} = n_{water}\sin\theta_{water} \quad \Rightarrow \sin\theta_{water} = \dfrac{n_{air}\sin\theta_{air}}{n_{water}}$

$$\Rightarrow \theta_{\text{red light, water}} = \sin^{-1}\left(\frac{n_{air}\sin\theta_{air}}{n_{\text{red light, water}}}\right) = \sin^{-1}\left(\frac{1(\sin 41.00^\circ)}{1.3311}\right) = \mathbf{29.53^\circ}.$$

(b) $$\theta_{\text{yellow light, water}} = \sin^{-1}\left(\frac{n_{air}\sin\theta_{air}}{n_{\text{yellow light, water}}}\right) = \sin^{-1}\left(\frac{1(\sin 41.00^\circ)}{1.3330}\right) = \mathbf{29.48^\circ}.$$

(c) Since $v = \dfrac{c}{n}$, the light that slows down the most is the light with the larger index of refraction, the **yellow light**.

28-11. (a) Apparent depth = actual depth$\left(\dfrac{n_{\text{air}}}{n_{\text{water}}}\right)$ = 10.0 cm$\left(\dfrac{1}{1.33}\right)$ = **7.5 cm.**

(b) The coin appears magnified in that it seems closer, so it occupies a larger angle in your visual field.

28-12. (a) The parallel rays of sunlight converge to the focal point of the lens. At this point the light is sufficiently concentrated to ignite the paper. So **the likely focal length of the lens is 15 cm.**

(b) At other distances the sunlight would not be sufficiently concentrated to ignite the paper.

28-13. (a) f = ? From $\dfrac{1}{d_o}+\dfrac{1}{d_i}=\dfrac{1}{f} \Rightarrow \dfrac{1}{f}=\left(\dfrac{d_i+d_o}{d_o d_i}\right) \Rightarrow f=\dfrac{d_o d_i}{d_i+d_o}=\dfrac{x(0.5x)}{x+0.5x}=\dfrac{0.5x^2}{1.5x}=\boldsymbol{\dfrac{x}{3}}.$

(b) $d_i=\dfrac{d_o f}{d_o-f}=\dfrac{2x\left(\frac{x}{3}\right)}{2x-\frac{x}{3}}=\dfrac{\frac{2x^2}{3}}{\frac{5x}{3}}=\boldsymbol{\tfrac{2}{5}x}.$

28-14. (a) $d_i=\dfrac{d_o f}{d_o-f}=\boldsymbol{\dfrac{xf}{x-f}}.$

(b) h_i = ? From $M=\dfrac{h_i}{h_o}=-\dfrac{d_i}{d_o} \Rightarrow h_i=-\dfrac{d_i}{d_o}h_o=-\dfrac{\left(\frac{xf}{x-f}\right)}{x}h_o=\boldsymbol{-\dfrac{fh_o}{x-f}}.$

(c) $d_i=\dfrac{xf}{x-f}=\dfrac{(7.0\text{ cm})(15\text{ cm})}{7.0\text{ cm}-15\text{ cm}}=$ **−13 cm**. Because d_i is negative, the image is virtual.

$h_i=-\dfrac{fh_o}{x-f}=-\dfrac{(15\text{ cm})(5.0\text{ cm})}{7.0\text{ cm}-15\text{ cm}}=$ **+9.4 cm**. Because h_i is positive, the image is right-side up.

(d) For f = −15 cm,

$d_i=\dfrac{xf}{x-f}=\dfrac{(7.0\text{ cm})(-15\text{ cm})}{7.0\text{ cm}-(-15\text{ cm})}=$ **−4.8 cm**, a virtual image.

$h_i=-\dfrac{fh_o}{x-f}=-\dfrac{(-15\text{ cm})(5.0\text{ cm})}{7\text{ cm}-(-15\text{ cm})}=$ **+3.4 cm**, a right-side up image.

28-15. (a) $d_i=\dfrac{d_o f}{d_o-f}=\dfrac{(25\text{ cm})(-33\text{ cm})}{25\text{ cm}-(-33\text{ cm})}=$ **−14 cm**, a virtual image.

(b) $M=\dfrac{h_i}{h_o}=-\dfrac{d_i}{d_o}=-\dfrac{(-14\text{ cm})}{25\text{ cm}}=$ **0.56.**

(c) Because the magnification is a positive number, **the image will be right-side up**.

28-16. (a) $d_i=\dfrac{d_o f}{d_o-f}=\dfrac{10.0\text{ m}(-0.045\text{ m})}{10.0\text{ m}-(-0.045\text{ m})}=$ **0.045 m = 45 mm**. This answer makes sense. Compared to the focal length of the lens, an object 10 meters away is essentially at infinity, so the image appears at the focal length of the lens.

(b) $M=\dfrac{h_i}{h_o}=-\dfrac{d_i}{d_o}=-\dfrac{(0.045\text{ m})}{10.0\text{ m}}=$ **−0.0045.**

28-17. (a) $d_0 = ?$ From $\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f} \Rightarrow \frac{1}{d_o}=\frac{1}{f}-\frac{1}{d_i}=\frac{d_i-f}{d_i f}$

$$\Rightarrow d_o = \frac{d_i f}{d_i - f} = \frac{(-22\text{ cm})(15\text{ cm})}{-22\text{ cm} - 15\text{ cm}} = \mathbf{8.9\text{ cm}}.$$

(b) $M = \frac{h_i}{h_o} = -\frac{d_i}{d_o} = -\frac{(-22\text{ cm})}{8.9\text{ cm}} = \mathbf{2.5}.$

(c) With the object within the focal length **you get an upright, magnified image**. If the object were located beyond the focal point the image would be upside down.

28-18. (a) $f = ?$ From $\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f} \Rightarrow \frac{1}{f}=\left(\frac{d_i+d_o}{d_o d_i}\right) \Rightarrow f=\frac{d_o d_i}{d_i+d_o}=\frac{(28\text{ cm})(3.0\text{ cm})}{3.0\text{ cm}+28\text{ cm}}=\mathbf{2.7\text{ cm}}.$

(b) The lens is a **converging lens** because only a converging lens can form real images.

28-19. (a) $f = ?$ From $\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f} \Rightarrow \frac{1}{f}=\left(\frac{d_i+d_o}{d_o d_i}\right) \Rightarrow f=\frac{d_o d_i}{d_i+d_o}=\frac{(1.9\text{ m})(0.098\text{ m})}{1.9\text{ m}+0.098\text{ m}}=\mathbf{0.093\text{ m}=9.3\text{ cm}}.$

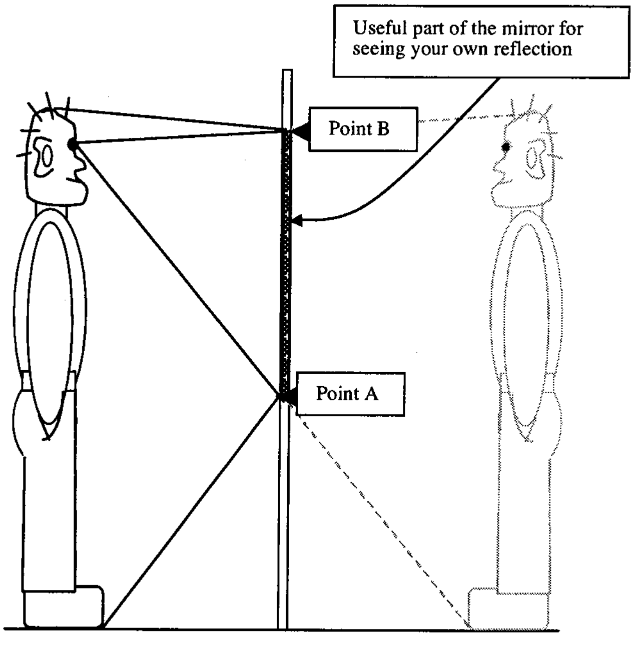
(b) $h_i = ?$ From $M=\frac{h_i}{h_o}=-\frac{d_i}{d_o} \Rightarrow h_i=-\frac{d_i}{d_o}h_o=-\frac{9.8\text{ cm}}{190\text{ cm}}(14\text{ cm})=-0.72\text{ cm}.$ The image is inverted and **0.72 cm in diameter.**

28-20. (a) $d_0 = ?$ From $\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f} \Rightarrow \frac{1}{d_o}=\frac{1}{f}-\frac{1}{d_i}=\frac{d_i-f}{d_i f}$

$$\Rightarrow d_o = \frac{d_i f}{d_i - f} = \frac{(-22\text{ cm})(2.2\text{ cm})}{-22\text{ cm} - 2.2\text{ cm}} = \mathbf{2.0\text{ cm}}.$$

(b) $M = \frac{h_i}{h_o} = -\frac{d_i}{d_o} = -\frac{(-22\text{ cm})}{2.0\text{ cm}} = \mathbf{11}.$

28-21. The diagram shows you, looking at your reflection in a full-length mirror. Light from your feet reflects symmetrically from the mirror at point A into your eyes. If we call the vertical distance between your eyes and the floor $h_{\text{eye-floor}}$, then the vertical distance between point A and your eyes is $\frac{1}{2}h_{\text{eye-floor}}$. (If you were to look at a point on the mirror below point A you'd see the reflection of the section of floor between your feet and the mirror.) Likewise, light from the top of your forehead reflects from point B into your eyes. If we call the vertical distance between your eyes and your forehead $h_{\text{eye-forehead}}$, then the vertical distance

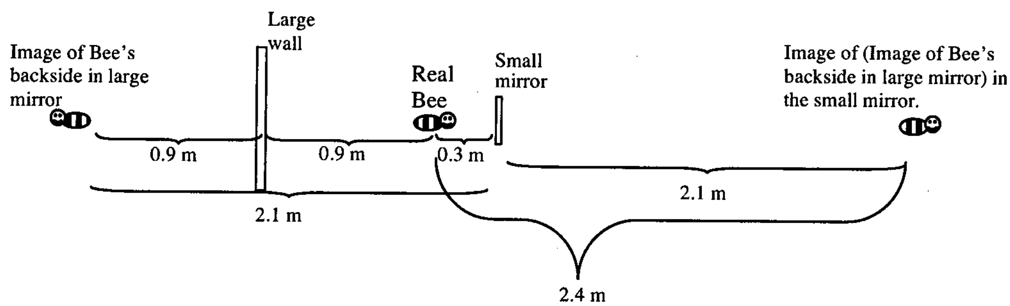


between point B and your eyes is $\frac{1}{2}h_{\text{eye-forehead}}$.

The vertical distance between point A and point B is $\frac{1}{2}h_{\text{eye-floor}} + \frac{1}{2}h_{\text{eye-forehead}} = \frac{1}{2}\left(h_{\text{eye-floor}} + h_{\text{eye-forehead}}\right) = \frac{1}{2}$(your height). This is the height of mirror that you need in order to be able to see your entire self (the front part, anyway).

Note that due to the symmetric nature of reflection, the vertical distance between Point A and your eyes will always be half of the vertical distance between the floor and your eyes, independent of how far away from the mirror you stand and the vertical distance between Point B and your eyes will always be half of the vertical distance between your forehead and your eyes. So regardless of how far way from the mirror you stand you'll still need a mirror only half your height to see all of you.

28-22. The diagram below may help. The bee's stinger is 0.9 m in front of the wall mirror, so the *image* of the stinger is 0.9 m behind the wall mirror. This *image* is 2.1 m in front of the small mirror, so the image of the image of the stinger will appear 2.1 m behind the small mirror. Since the small mirror is 0.3 m from the bee's face, the image of the image of the stinger appears 2.4 m in front of the bee's face.



28-23. $$\frac{v_{\text{light in diamond}}}{v_{\text{light in air}}} = \frac{\left(\dfrac{c}{n_{\text{diamond}}}\right)}{\left(\dfrac{c}{n_{\text{air}}}\right)} = \frac{n_{\text{air}}}{n_{\text{diamond}}} = \frac{1}{2.42} = \mathbf{0.413 \approx 41\%}.$$

28-24. $$\frac{\sin\theta_1}{\sin\theta_2} = \frac{v_1}{v_2} = \frac{\frac{c}{n_1}}{\frac{c}{n_2}} = \frac{n_2}{n_1} \Rightarrow n_1 \sin\theta_1 = n_2 \sin\theta_2.$$

28-25. From $n_{\text{air}} \sin\theta_{\text{air}} = n_{\text{glass}} \sin\theta_{\text{glass}} \Rightarrow \sin\theta_{\text{glass}} = \dfrac{n_{\text{air}} \sin\theta_{\text{air}}}{n_{\text{glass}}}$

$$\Rightarrow \theta_{\text{glass}} = \sin^{-1}\left(\frac{n_{\text{air}} \sin\theta_{\text{air}}}{n_{\text{glass}}}\right) = \sin^{-1}\left(\frac{1(\sin 37.0°)}{1.52}\right) = \mathbf{23.3°}.$$

28-26. From $n_{\text{air}}\sin\theta_{\text{air}} = n_{\text{glass}}\sin\theta_{\text{glass}} \quad \Rightarrow \sin\theta_{\text{glass}} = \dfrac{n_{\text{air}}\sin\theta_{\text{air}}}{n_{\text{glass}}}$

$$\Rightarrow \theta_{\text{glass}} = \sin^{-1}\left(\frac{n_{\text{air}}\sin\theta_{\text{air}}}{n_{\text{glass}}}\right) = \sin^{-1}\left(\frac{1(\sin 45°)}{1.50}\right) = \mathbf{28°}.$$

28-27. From $n_{\text{water}}\sin\theta_{\text{water}} = n_{\text{air}}\sin\theta_{\text{air}} \quad \Rightarrow \sin\theta_{\text{water}} = \dfrac{n_{\text{air}}\sin\theta_{\text{air}}}{n_{\text{water}}} \Rightarrow \theta_{\text{water}} = \sin^{-1}\left(\dfrac{n_{\text{air}}\sin\theta_{\text{air}}}{n_{\text{water}}}\right).$

The *critical angle* is the angle the incident light beam makes with the normal in the water that would result in the beam being refracted through an angle of 90° in the air. (It's called the *critical* angle because beyond this angle there is no more air to refract into, so the beam is totally reflected off of air/water interface back into the water). Setting $\theta_{\text{air}} = 90°$ we get

$$\Rightarrow \theta_{\text{water}} = \sin^{-1}\left(\frac{1(\sin 90°)}{1.33}\right) = \mathbf{48.8°}.$$

28-28. From $n_{\text{diamond}}\sin\theta_{\text{diamond}} = n_{\text{air}}\sin\theta_{\text{air}} \quad \Rightarrow \sin\theta_{\text{diamond}} = \dfrac{n_{\text{air}}\sin\theta_{\text{air}}}{n_{\text{diamond}}} \Rightarrow \theta_{\text{diamond}} = \sin^{-1}\left(\dfrac{n_{\text{air}}\sin\theta_{\text{air}}}{n_{\text{diamond}}}\right)$

$$\Rightarrow \theta_{\text{diamond}} = \sin^{-1}\left(\frac{1(\sin 90°)}{2.42}\right) = \mathbf{24.4°}.$$

28-29. f = ? From $\dfrac{1}{d_o} + \dfrac{1}{d_i} = \dfrac{1}{f} \quad \Rightarrow \dfrac{1}{f} = \left(\dfrac{d_i + d_o}{d_o d_i}\right) \quad \Rightarrow f = \dfrac{d_o d_i}{d_i + d_o} = \dfrac{(40.0\,\text{cm})(80.0\,\text{cm})}{80.0\,\text{cm} + 40.0\,\text{cm}} = 26.7\,\text{cm} \approx \mathbf{27\ cm.}$

28-30. f = ? From $\dfrac{1}{d_o} + \dfrac{1}{d_i} = \dfrac{1}{f} \quad \Rightarrow \dfrac{1}{f} = \left(\dfrac{d_i + d_o}{d_o d_i}\right) \quad \Rightarrow f = \dfrac{d_o d_i}{d_i + d_o} = \dfrac{(15\,\text{cm})(30\,\text{cm})}{30\,\text{cm} + 15\,\text{cm}} = \mathbf{10\ cm.}$

28-31. d_i = ? The image will be sharp on the screen if the screen is located at the image distance.

$$d_i = \frac{d_o f}{d_o - f} = \frac{2.5\text{ m }(0.300\text{ m})}{2.5\text{ m} - 0.300\text{ m}} = 0.34\text{ m} = \mathbf{34\ cm.}$$

28-32. $d_i = \dfrac{d_o f}{d_o - f} = \dfrac{16\text{ cm }(10\text{ cm})}{16\text{ cm} - 10\text{ cm}} = 26.7\text{ cm} \approx \mathbf{27\ cm.}$

28-33. $d_i = \dfrac{d_o f}{d_o - f} = \dfrac{16\text{ cm }(-10\text{ cm})}{16\text{ cm} - (-10\text{ cm})} = \mathbf{-6.2\ cm.}$

28-34. $M = -\dfrac{d_i}{d_o} = -\dfrac{\left(\frac{d_o f}{d_o - f}\right)}{d_o} = -\dfrac{f}{d_o - f} = -\dfrac{10.0\text{ cm}}{50.0\text{ cm} - 10.0\text{ cm}} = \mathbf{-0.25}.$

28-35. $d_o = ?$ Because the lens is converging we know that f is positive. Because the image is real we know that d_i is positive.

From $\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f} \Rightarrow \frac{1}{d_o}=\frac{1}{f}-\frac{1}{d_i}=\frac{d_i-f}{d_i f} \Rightarrow d_o=\frac{d_i f}{d_i-f}=\frac{(200\,\text{cm})(20\,\text{cm})}{200\,\text{cm}-20\,\text{cm}}=\mathbf{22\ cm.}$

28-36. $h_i = ?$ From $M=\frac{h_i}{h_o}=-\frac{d_i}{d_o} \Rightarrow h_i=-\frac{d_i}{d_o}h_o$. Since we don't know if the image is real or not (equivalently, whether d_i is positive or negative) we use absolute values:

$|h_i|=\left|\frac{d_i}{2d_i}\right|5.0\text{ cm}=\frac{1}{2}(5.0\text{ cm})=\mathbf{2.5\ cm.}$

28-37. The image between the lens and the screen is the image distance, d_i. Since the image is real (it forms on a screen) it will be inverted and h_i will be negative.

From $M=\frac{h_i}{h_o}=-\frac{d_i}{d_o} \Rightarrow d_i=-\frac{h_i}{h_o}d_o=-\frac{-1.0\text{ cm}}{200\text{ cm}}(8.0\text{ m})=\mathbf{0.040\ m=4.0\ cm.}$

28-38. $d_i=\frac{d_o f}{d_o-f}=\frac{30\text{ cm }(20\text{ cm})}{30\text{ cm}-20\text{ cm}}=\mathbf{60\ cm.}$ Note that neither the index of refraction of the glass nor the height of the candle come into the equation.

28-39. $f = ?$ "Width" is just "height" turned sideways. We can use the same magnification equation for width as we can for height. Since the image from a single diverging lens is always right-side up, h_i (or w_i) will be positive.

From $M=\frac{h_i}{h_o}=-\frac{d_i}{d_o} \Rightarrow d_i=-\frac{h_i}{h_o}d_o=-\frac{\frac{1}{5}(5.0\text{ cm})}{5.0\text{ cm}}(10.0\text{ cm})=-2.0\text{ cm.}$

From $\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f} \Rightarrow \frac{1}{f}=\left(\frac{d_i+d_o}{d_o d_i}\right) \Rightarrow f=\frac{d_o d_i}{d_i+d_o}=\frac{(10.0\,\text{cm})(-2.0\,\text{cm})}{-2.0\,\text{cm}+10.0\,\text{cm}}=\mathbf{-2.5\ cm.}$

28-40. $f = ?$ Since the magnification is positive d_i will be negative.

From $M=-\frac{d_i}{d_o} \Rightarrow d_i=-Md_o=-(5.00)(5.00\text{ cm})=-25.0\text{ cm.}$

From $\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f} \Rightarrow \frac{1}{f}=\left(\frac{d_i+d_o}{d_o d_i}\right) \Rightarrow f=\frac{d_o d_i}{d_i+d_o}=\frac{(5.00\,\text{cm})(-25.0\,\text{cm})}{-25.0\,\text{cm}+5.0\,\text{cm}}=\mathbf{-6.25\ cm.}$