Trig Functions

Degrees/Radians

1 rotation = 360° or 2π radians. Radians to degrees -multiply by $\frac{180^\circ}{\pi}$. Degrees to radian - multiply by $\frac{\pi}{180^\circ}$

Trig Ratios

$\sin\theta = \frac{opp}{hyp}$ $\cos\theta = \frac{adj}{hyp}$ $\tan\theta = \frac{opp}{adj}$ $\csc\theta = \frac{hyp}{opp}$ $\sec\theta = \frac{hyp}{adj}$ $\cot\theta = \frac{adj}{opp}$

Reciprocal Identities

$\sin\theta = \frac{1}{\csc\theta}$ $\cos\theta = \frac{1}{\sec\theta}$ $\tan\theta = \frac{1}{\cot\theta}$ $\csc\theta = \frac{1}{\sin\theta}$ $\sec\theta = \frac{1}{\cos\theta}$ $\cot\theta = \frac{1}{\tan\theta}$

Quotient Identities

$\tan\theta = \frac{\sin\theta}{\cos\theta}$ $\cot\theta = \frac{\cos\theta}{\sin\theta}$

Pythagorean Identities

$\sin^2\theta + \cos^2\theta = 1$ $1 + \tan^2\theta = \sec^2\theta$ $1 + \cot^2\theta = \csc^2\theta$

Cofunction Identities

$\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta$ $\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$ $\tan\left(\frac{\pi}{2} - \theta\right) = \cot\theta$ $\cot\left(\frac{\pi}{2} - \theta\right) = \tan\theta$ $\sec\left(\frac{\pi}{2} - \theta\right) = \csc\theta$ $\csc\left(\frac{\pi}{2} - \theta\right) = \sec\theta$

Odd/Even Identities

$\sin(-\theta) = -\sin\theta$ $\cos(-\theta) = \cos\theta$ $\tan(-\theta) = -\tan\theta$ $\csc(-\theta) = -\csc\theta$ $\sec(-\theta) = \sec(\theta)$ $\cot(-\theta) = -\cot\theta$

Unit Circle

In the unit circle, you can find the value of a function at a given angle, θ radians, by finding the coordinates of the point on the circle intercepted by a ray θ radians from the initial side. Remember, the x coordinate is equal to the $\cos\theta$ and the y coordinate is equal to the $\sin\theta$. Remember, your identities to find the others.

Sum/Difference Identities

$$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta \qquad \cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta \qquad \tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$$

$$\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta \qquad \cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta \qquad \tan(\alpha-\beta)=\frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$$

Double-angle Formulas

$$\sin 2\theta=2\sin\theta\cos\theta \qquad \cos 2\theta=\cos^2\theta-\sin^2\theta \qquad \cos 2\theta=1-2\sin^2\theta \qquad \cos 2\theta=2\cos^2-1 \qquad \tan 2\theta=\frac{2\tan\theta}{1-\tan^2\theta}$$

Half-Angle Formulas

$$\sin\left(\frac{\theta}{2}\right)=\pm\sqrt{\frac{1-\cos\theta}{2}} \qquad \cos\left(\frac{\theta}{2}\right)=\pm\sqrt{\frac{1+\cos\theta}{2}} \qquad \tan\left(\frac{\theta}{2}\right)=\frac{1-\cos\theta}{\sin\theta}=\frac{\sin\theta}{1+\cos\theta}$$

Power-Reducing Formulas

$$\sin^2\theta=\frac{1-\cos 2\theta}{2} \qquad \cos^2\theta=\frac{1+\cos 2\theta}{2} \qquad \tan^2\theta=\frac{1-\cos 2\theta}{1+\cos 2\theta}$$

Sum-to-Product Formulas

$$\sin\alpha+\sin\beta=2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right) \qquad \cos\alpha+\cos\beta=2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

$$\sin\alpha-\sin\beta=2\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right) \qquad \cos\alpha-\cos\beta=-2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$$

Product-to-Sum Formulas

$$\sin\alpha\sin\beta=\frac{1}{2}\left[\cos(\alpha-\beta)-\cos(\alpha+\beta)\right] \qquad \sin\alpha\cos\beta=\frac{1}{2}\left[\sin(\alpha+\beta)+\sin(\alpha-\beta)\right]$$

$$\cos\alpha\cos\beta=\frac{1}{2}\left[\cos(\alpha-\beta)+\cos(\alpha+\beta)\right] \qquad \cos\alpha\sin\beta=\frac{1}{2}\left[\sin(\alpha+\beta)-\sin(\alpha-\beta)\right]$$

Law of Sines

$$\frac{\text{a}}{\sin\text{A}}=\frac{\text{b}}{\sin\text{B}}=\frac{\text{c}}{\sin\text{C}} \qquad \frac{\sin\text{A}}{\text{a}}=\frac{\sin\text{B}}{\text{b}}=\frac{\sin\text{C}}{\text{c}}$$

Law of Cosines

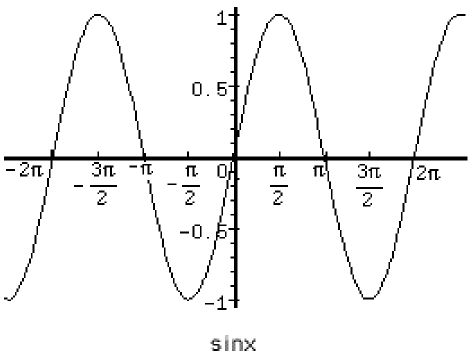
$$a^2=b^2+c^2-2bc\cos A \qquad b^2=a^2+c^2-2ac\cos B \qquad c^2=a^2+b^2-2ab\cos C$$

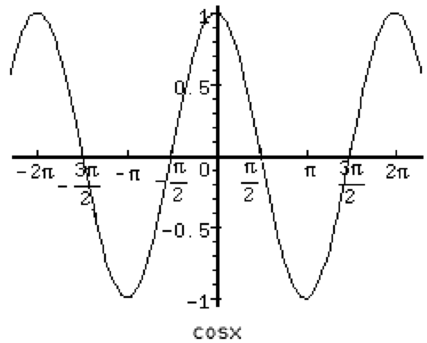
Graphs of Trig Functions

Symmetry

$y = \sin\theta$ is symmetric with respect to the origin. It is an odd function.
$y = \cos\theta$ is symmetric with respect to the y axis. It is an even function.
$y = \tan\theta$ is symmetric with respect to the origin. It is an odd function.

Sine and Cosine





Standard form: $y = A\sin(Bx - C) + D$ $y = A\cos(Bx - C) + D$, period of plain sin x and cos x is 2π.

The amplitude is $|A|$, the period is $\dfrac{2\pi}{|B|}$, $\dfrac{C}{B}$ determines horizontal shifts and D determines vertical shifts.

*******To find the B you can factor first,*** $y = \sin(2x - \pi)$ *can be factored to* $y = \sin 2\left(x - \dfrac{\pi}{2}\right)$.

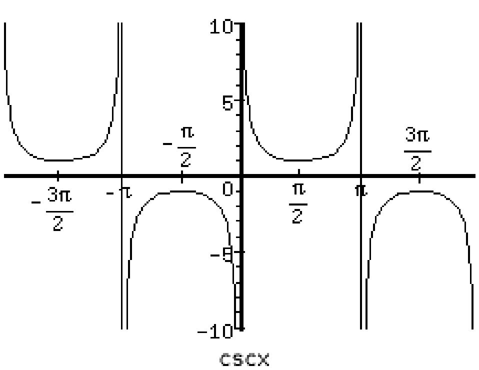
*This will give you a period of π and a shift of π/2 to the right.****

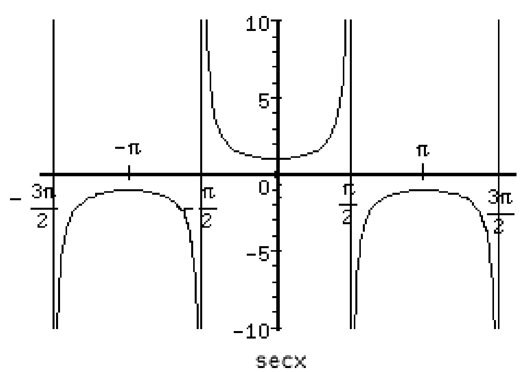
Sin goes up through the origin to a maximum of 1 at π/2. Then back down through the x axis (zero) at π to a minimum of -1 at 3π/2 and back up through the x axis (zero) at 2π. As it goes to the left it goes down so it has a minimum of -1 at -π/2, back up through the x axis (zero) -π and so on. It is helpful to track the points it crosses the x axis (zeros) when the period changes. Since there is a zero at the origin there will also be a zero every period. Since it also comes back through the x axis half way between the origin and one period there will also be a zero at every 1/2 period. If the period changes (shrinks or stretches) you can still apply these concepts once you have determined the new period. You must use the amplitude and shifts to come up with the final graph.

Cos has a maximum on the y axis. Since it is the same shape (just shifted) as sin it will have zeros an equal distance from the y axis. This distance will be 1/4 a period. This is because it is 1/2 period from the max to the min and the zero will be half way between them. Cos will then have a zero every 1/2 period from there is both directions. If the period changes (shrinks or stretches) you can still apply these concepts once you have determined the new period. You must use the amplitude and shifts to come up with the final graph.

The domain for sin and cos will be $-\infty$ to ∞. The range will be from the minimum to the maximum (this can be determined from the amplitude and vertical shift).

Secant and Cosecant





Standard form: $y = A\csc(Bx - C) + D$ $y = A\sec(Bx - C) + D$, period of plain csc x and sec x is 2π.

The amplitude is $|A|$, the period is $\frac{2\pi}{|B|}$, $\frac{C}{B}$ determines horizontal shifts and D determines vertical shifts.

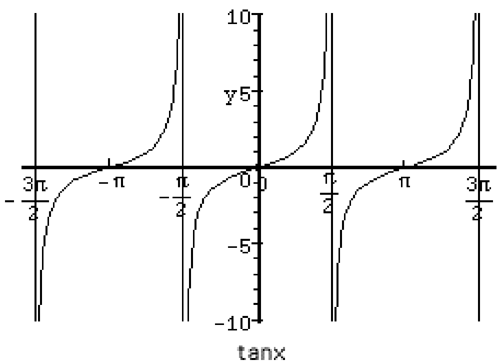
Remember, the graphs of $y = \sec\theta$ and $y = \csc\theta$ actually have two pieces (a top and a bottom) each period. Therefore, they have an asymptote half way through each period.

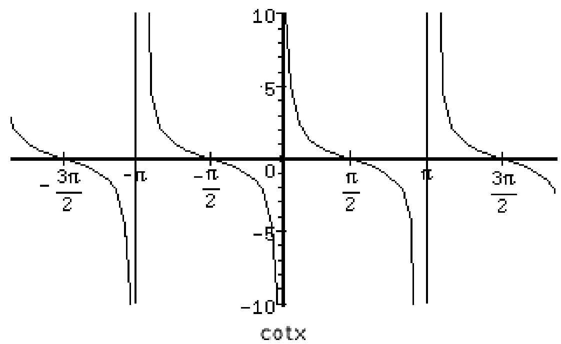
Graphing sec, which is the reciprocal of cos. Graph the corresponding cos graph first. From this graph you should be able to find the reciprocal of the points. Remember, there will be an asymptote on the sec graph whenever the cos graph is zero (there is no reciprocal of zero).

Graphing csc, which is the reciprocal of sin. Graph the corresponding sin graph first. From this graph you should be able to find the reciprocal of the points. Remember, there will be an asymptote on the csc graph whenever the sin graph is zero (there is no reciprocal of zero).

The domain for sec and csc will be all real numbers except where there are asymptotes. The range will be $-\infty$ to the highest point of the bottom part of the graph and the minimum of the top part to ∞.

Tangent and Cotangent





Standard form: $y = A\tan(Bx - C) + D$ $y = A\cot(Bx - C) + D$, period of plain tan x and cot x is π

The period is $\frac{\pi}{|B|}$, $\frac{C}{B}$ determines horizontal shifts and D determines vertical shifts.

Remember $\tan\theta = \frac{\sin\theta}{\cos\theta}$, therefore it has asymptotes when $\cos\theta = 0$ (every multiple of π/2) and zeros when $\sin\theta = 0$ (every multiple of π). Tan will have asymptotes equal distance from the y axis, 1/2 a period on each side. It will then have an asymptote every period in both directions. It will have an x intercept half way between each set of asymptotes. If the period changes (shrinks or stretches) you can still apply these concepts once you have determined the new period.

Remember $\cot\theta = \frac{\cos\theta}{\sin\theta}$, therefore it has asymptotes when $\sin\theta = 0$(every multiple of π) and zeros when $\cos\theta = 0$ (every multiple of π/2). Cot will have an asymptote at x=0, $\sin 0 = 0$, and then every period in both directions. It will have an x intercept half way between each set of asymptotes. If the period changes (shrinks or stretches) you can still apply these concepts once you have determined the new period.

The domain for tan /cot will be all real numbers except where there are asymptotes. The range will be $-\infty$ to ∞.