

Curriculum Compacting-Encoding/Decoding Messages with Matrix Multiplication.

Name _____

Encoding/Decoding a message.

1. Think of a short message you would like to 'encode'. It should not be too long.
2. Convert the message to a series of numbers using the following key.

_ =0	E=5	J=10	O=15	T=20	Y=25
A=1	F=6	K=11	P=16	U=21	Z=26
B=2	G=7	L=12	Q=17	V=22	
C=3	H=8	M=13	R=18	W=23	
D=4	I=9	N=14	S=19	X=24	

For example, 'secret message' would be the numbers 19 5 3 18 5 20 0 13 5 19 19 1 7 5

3. Now break the series of numbers into 1 x 2 matrices. For example, the numbers for 'secret message' would be represented by the following matrices $\begin{bmatrix} 19 & 5 \end{bmatrix}$ $\begin{bmatrix} 3 & 18 \end{bmatrix}$ $\begin{bmatrix} 5 & 20 \end{bmatrix}$ $\begin{bmatrix} 0 & 13 \end{bmatrix}$ $\begin{bmatrix} 5 & 19 \end{bmatrix}$ $\begin{bmatrix} 19 & 1 \end{bmatrix}$ $\begin{bmatrix} 7 & 5 \end{bmatrix}$.
4. Now pick a 2 x 2 matrix to use as your encoding matrix. This matrix **MUST** have an inverse.

5. Multiply each of the 1 x 2 matrices by your 2 x 2 encoding matrix to get a series of encoded matrices.

$$\begin{bmatrix} 19 & 5 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 3 & 4 \end{bmatrix}, \begin{bmatrix} 3 & 18 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 3 & 4 \end{bmatrix}, \begin{bmatrix} 5 & 20 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 3 & 4 \end{bmatrix}, \text{etc.}$$

6. Encoded matrices would be $\begin{bmatrix} 34 & 39 \end{bmatrix}$ $\begin{bmatrix} 57 & 75 \end{bmatrix}$ $\begin{bmatrix} 65 & 85 \end{bmatrix}$ etc.

7. Using these matrices and the key above you can create an 'encoded' message.

8. To decode a message you would need to multiply each of the 1 x 2 encoded matrices by the inverse of the 2 x 2 encoding matrix. $\begin{bmatrix} 34 & 39 \end{bmatrix} \begin{bmatrix} 4 & -1 \\ -3 & 1 \end{bmatrix}, \begin{bmatrix} 57 & 75 \end{bmatrix} \begin{bmatrix} 4 & -1 \\ -3 & 1 \end{bmatrix}, \begin{bmatrix} 65 & 85 \end{bmatrix} \begin{bmatrix} 4 & -1 \\ -3 & 1 \end{bmatrix}, \text{etc.}$

9. This will get you back to the original matrices $\begin{bmatrix} 19 & 5 \end{bmatrix}$ $\begin{bmatrix} 3 & 18 \end{bmatrix}$ $\begin{bmatrix} 5 & 20 \end{bmatrix}$ $\begin{bmatrix} 0 & 13 \end{bmatrix}$ $\begin{bmatrix} 5 & 19 \end{bmatrix}$ $\begin{bmatrix} 19 & 1 \end{bmatrix}$ $\begin{bmatrix} 7 & 5 \end{bmatrix}$. Using the key you can then see what the actual message is.

Assignment.

1. Think of a short message.
2. Create an 'encoding' matrix and encode your message.
3. When you turn in your message, you will receive an encoded message along with the matrix that was used to encode it.
4. Decode the message given to you and explain how and why this process works.